Übungen zu Analysis III

Blatt 5

1 Prove Remark 8.3.13. [4]

2 Let $\Omega \subset \mathbb{R}^n$, $f \in C^1(\Omega, \mathbb{R}^n)$ and assume $Df(x) \in GL(n)$ for all $x \in \Omega$, then the function $\varphi(x) = |f(x)|$ doesn't attain a maximum in Ω. [2]

3 Let $f \in C^1(\mathbb{R}^n, \mathbb{R}^n)$ such that $\|Df\| \le \kappa < 1$ and define

(i) $g(x) = x + f(x) \qquad \forall\, x \in \mathbb{R}^n$,

(ii) $h(x,y) = (x + f(y), y + f(x)) \qquad \forall\, (x,y) \in \mathbb{R}^n \times \mathbb{R}^n$,

then g and h are surjective. [6]

4 Prove that the equations

$$x^2 + y^2 - \xi^2 - \eta = 0,$$

$$x^2 + 2y^2 + 3\xi^2 + 4\eta^2 = 1$$

can be solved for (ξ, η) in a neighbourhood of $(x, y, \xi, \eta) = (\frac{1}{2}, 0, \frac{1}{2}, 0)$. Determine the first derivatives of (ξ, η). [4]