

Workshop

Computability and Models

18th - 20th January 2001
University of Heidelberg



Programme Committee

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PROGRAM

Thursday, January 18, 2001

Session 1 (Chair: Klaus Ambos-Spies)

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14.10 - 15.00	Sergei S. Goncharov (Novosibirsk) <i>Computable Numberings and Rogers Structures</i>
15.00 - 15.30	COFFEE
15.30 - 16.00	Serikzhan Badaev (Alma-Ata) <i>Spectrum of Computable Minimal Numberings</i>
16.00 - 16.30	Oleg V. Kudinov (Novosibirsk) <i>Complexity of Problems in Computable Model Theory</i>
16.30 - 17.00	Frank Stephan (Heidelberg) <i>Learning to Separate Recursive Sets</i>
17.15	Business Meeting on the INTAS-RFBR Project COMPUTABILITY AND MODELS
19.30	DINNER Restaurant ROSE, Handschuhsheimer Landstr. 130

Friday, January 19, 2001

Session 2 (Chair: Sergey S. Goncharov)

09.00 - 09.50	Marat Arslanov (Kazan) <i>Models of Relative Computability and the Ershov Hierarchy</i>
10.00 - 10.30	Angsheng Li (Leeds) <i>Splitting and Nonsplitting in the Computably Enumerable Degrees</i>
10.30 - 11.00	COFFEE
11.00 - 11.50	Steffen Lempp (Madison) <i>On Computable Self-Embeddings of Computable Linear Orders</i>

11.50 - 12.20 Iskander Kalimullin (Kazan)
On the Group Generated by Primitive Recursive Permutations

12.20 - 14.00 LUNCH

Session 3 (Chair: S. Barry Cooper)

14.00 - 14.30 Andre Nies (Chicago)
Effectively Dense Boolean Algebras and Their Applications

14.30 - 15.00 Andrei S. Morozov (Novosibirsk)
On Sigma-Definability of Admissible Sets as a Reducibility

15.00 - 15.30 Vasco Brattka (Hagen)
Recursive Subsets of the Real Numbers

15.30 - 16.00 COFFEE

16.00 - 16.30 Jan Reimann (Heidelberg)
Resource-Bounded Hausdorff Dimension

16.30 - 17.00 Mikhail Peretyatkin (Alma-Ata)
Algorithmic Nature of Classical First-Order Predicate Logic

Saturday, January 20, 2001

Session 4 (Chair: Marat Arslanov)

09.30 - 10.20 William Gasarch (College Park)
Bounded Queries in Computability Theory

10.20 - 11.00 COFFEE

11.00 - 11.30 Eberhard Herrmann (Berlin)
r-Maximal Sets

11.30 - 12.00 Boris Solon (Ivanovo)
Characterising the Non-Total e-Degrees

12.00 LUNCH

ABSTRACTS

Models of Relative Computability and the Ershov Hierarchy

Marat Arslanov

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One focus of my talk will be the n -r.e. hierarchy of sets and the corresponding degree structure. A particular focus will be the definability of the various levels of the n -r.e. hierarchy, both relatively and within wider local structures; more specifically, questions related to the definability of the relations of ‘recursively enumerable’ and ‘recursively enumerable in’ in the degree structures of the n -r.e. degrees for $n > 1$ (which would allow one to obtain a negative solution to the conjecture concerning the elementary equivalence of the different levels of the n -r.e. hierarchy).

In second part of my talk I will consider general conditions under which relative splittings and specified diamond embeddings are derivable in the local structure of the enumeration degrees. General questions of definability and the role of splitting and nonsplitting, and also the description of new relationships between information content and degree theoretic structure will be considered.

Spectrum of Computable Minimal Numberings

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We consider computable minimal numberings of the families of arithmetical sets (see [1] for the necessary notions).

Let $\mathcal{A} \subseteq \Sigma_n^0$ be any Σ_n^0 -computable family of sets. Let $\langle d, p, m \rangle$, $d, p, m \in \omega \cup \{\omega\}$, be respectively number up to equivalence of decidable, positive non-decidable, and minimal non-positive Σ_n^0 -computable minimal numberings of \mathcal{A} . The triple $\langle d, p, m \rangle$ is called *spectrum of computable minimal numberings* of \mathcal{A} .

The problem of description of the triples which realize spectrums of computable minimal numberings for the families in a given level of hierarchy arose from the long-standing problem of Yu.L.Ershov on possible number of computable minimal numberings of the families of c.e. sets. Note that not every triple could be a spectrum of some family of c.e. sets. This follows from the following theorem of S.S.Goncharov [2]: if a family of c.e. sets has decidable but not the least computable numbering under reducibility then it has ω positive computable numberings.

It is well known that the parameter d of spectrums of computable families of c.e. sets is closely connected to the algorithmic dimension of decidable models [3].

In opposite to the classical case of computable families of c.e. sets, spectrum of Σ_{n+2}^0 -computable minimal numberings is completely described in [4] as follows. For every Σ_{n+2}^0 -computable family \mathcal{A} , its spectrum of Σ_{n+2}^0 -computable numberings is either $\langle \omega, \omega, \omega \rangle$ or $\langle 0, 0, \omega \rangle$. Both of these spectrums are realized.

References

- [1] S.S.Goncharov, A.Sorbi, *Generalised computable numerations and non-trivial Rogers semilattices*. Algebra and Logic, 1997, vol. 36, n. 4, pp. 359–369.
- [2] S.S.Goncharov, *Positive numberings of the families with one-to-one numberings*. Algebra and Logic, 1983, vol. 22, n. 5, pp. 481–488.
- [3] S.S.Goncharov, *Problem of a number of nonautoequivalent constructivizations*. Algebra and Logic, 1980, vol. 19, n. 6, pp. 621–639.
- [4] S.A.Badaev, S.S.Goncharov, *On Rogers semilattices of the families of arithmetical sets*. Algebra and Logic, 2000, vol. 39 (to appear).

Recursive Subsets of the Real Numbers

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In his book “The Emperor’s New Mind” Roger Penrose implicitly defines some criteria which should be met by a reasonable notion of recursiveness for subsets of Euclidean space. We discuss two such notions with regard to Penrose’s criteria: one originated from computable analysis, the other one introduced by Blum, Shub and Smale.

References

- [1] Lenore Blum, Felipe Cucker, Michael Schub, and Steve Smale. *Complexity and Real Computation*. Springer, New York, 1998.
- [2] Vasco Brattka, *Recursive and Computable Operations over Topological Structures*, PhD Thesis, FernUniversität Hagen, 1999,
available: www.informatik.fernuni-hagen.de/thi1/vasco.brattka/thesis/
- [3] Vasco Brattka and Klaus Weihrauch. Computability on subsets of Euclidean space I: Closed and compact subsets. *Theoretical Computer Science*, 219:65–93, 1999.
- [4] Roger Penrose. *The Emperor’s New Mind. Concerning Computers, Minds and The Laws of Physics*. Oxford University Press, New York, 1989.
- [5] Klaus Weihrauch, *Computable Analysis*, Springer, Berlin, 2000

A Survey of Bounded Queries in Computability Theory

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In the mid 1980's Richard Beigel and William Gasarch began studying the complexity of functions and sets in terms of the *number of questions* needed to compute them. In the last 15 years other researchers, notably Martin Kummer and Frank Stephan, have also done research on this topic. Over time two types of questions emerged:

1. For several natural functions $f \leq_T X$ *exactly how many queries to X are required?*
Two examples:
 - (a) To compute which of x_1, \dots, x_n is in K , can be solved with far fewer than n queries.
 - (b) To compute which of x_1, \dots, x_n is in TOT , requires n queries.
2. For various sets A , *are n queries to A more powerful than $n - 1$?* Two examples:
 - (a) For all n , there are functions that can be computed with n queries to K that cannot be computed with $n - 1$ queries to K .
 - (b) Let B be a hyperimmune-free set and let $A = B^{tt}$. If $X \leq_T A$ then X can be computed with 1 query to A .

In this talk we will discuss the following

1. What motivated these types of questions?
2. What results are known?
3. What techniques were used to obtain these results?
4. What questions are open?

Computable Numberings and Rogers Structures

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It is suggested a general approach to the notion of computable numberings for the wide variety of the classes of constructive objects. We consider mainly the problems related to algebraic properties for Rogers semilattices of computable numberings and some results about initial segments of Rogers semilattices for arithmetic classes and about covers in these semilattices. We will discuss some problems about Rogers semilattices for Ershov hierarchies, too.

r-Maximal Sets

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The r-maximal sets and their properties were investigated in several papers. Here it will be given a systematical presentation of all these results including also few which are still not published. In the centre of interest are the atomless r-maximal sets and the different methods of constructing them. In particular the already known lattice properties of the r-maximal sets are treated. But also degree properties of them and more general of the r-cohesive sets are given. Further index set estimations of special classes of r-maximal sets and the relationship between classes of r-maximal sets and other classes of c.e. sets are considered.

On the Group Generated by Primitive Recursive Permutations

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Let \mathcal{C} be the group of all computable permutations of ω . Let $\mathcal{P} \subset \mathcal{C}$ be the class of all primitive recursive permutations. Consider the subgroup $\langle \mathcal{P} \rangle$. It is the least subgroup containing \mathcal{P} . We prove that $\langle \mathcal{P} \rangle = \mathcal{C}$. To obtain this result we consider the class $\mathcal{P}_1 \subseteq \langle \mathcal{P} \rangle$ of computable permutations which are presentable as $\pi\tau^{-1}$, where π and τ are primitive recursive permutations. We also study the class $\mathcal{P}_2 \subseteq \langle \mathcal{P} \rangle$ of functions which are presentable as a superposition of two permutations from \mathcal{P}_1 .

It is shown that each computable permutation can be presented as a superposition of a permutation from \mathcal{P}_1 and a permutation from \mathcal{P}_2 .

Complexity of Problems in Computable Model Theory and Categoricity

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For any countable model \mathbf{A} it is natural to consider the set of Turing degrees $\text{dg}(\mathbf{A}) = \{a \mid \text{for some model } \mathbf{B} \cong \mathbf{A} \text{ with domain } |\mathbf{B}| \subseteq \omega \text{ the atomic diagram of } (\mathbf{B}, b)_{b \in |\mathbf{B}|} \text{ has a degree } a\}$.

It is well-known that this set does not necessary possess the least degree. This fact leads us to the following natural Goncharov's problem. Do there exist Turing degrees $a|b$ that for a suitable countable model \mathbf{A} $\text{dg}(\mathbf{A}) = \{d \mid d \geq a \vee d \geq b\}$?

This problem is still open but for c.e. degrees the answer is negative.

Theorem 1 *For any countable model \mathbf{A} its $\text{dg}(\mathbf{A})$ cannot be presented in the form $\{d \mid d \geq a \vee d \geq b\}$ for some incomparable c.e. degrees a, b .*

To take into account a difference between nonequivalent presentations of given model \mathbf{A} , one can define a function $Sp_{\mathbf{A}}(\mathbf{A}) : \mathbf{D} \rightarrow \omega \cup \{\omega\}$ by the rule: $Sp_{\mathbf{A}}(\mathbf{A})(d)$ is equal to quantity of presentations of \mathbf{A} of degree d up to equivalence being defined via computable in d isomorphisms between presentations. In particular, $d \in \text{dg}(\mathbf{A})$ iff $Sp_{\mathbf{A}}(\mathbf{A})(d) > 0$.

Remark 1 *A computable model \mathbf{A} is relatively categorical iff it possesses a computable Scott family of existential formulas (with finite number of parameters) iff $\forall d Sp_{\mathbf{A}}(\mathbf{A})(d) = 1$.*

There are some known restrictions on functions $Sp_{\mathbf{A}}$ – the problem is to describe them.

As for the problem of existence of a computable copy for given model, it is reduced to similar problem for the computable fragment of $L_{\omega_1, \omega}$.

Theorem 2 *A hyperarithmetical model \mathbf{M} has a computable copy iff for every constructive ordinal α the set of realized in \mathbf{M} computable infinitary sentences of rank bounded by α has a computable model.*

To investigate important classes of models, let us fix any natural computable enumeration ψ of partial atomic diagrams of computable models in the language without functional symbols. The index set of the class of computable models defined via ψ measures its complexity.

Theorem 3 *The index set of the class of categorical models is m -complete in Π_1^1 . The index set of the class of categorical 1-decidable models is m -complete in Π_4^0 .*

On Computable Self-Embeddings of Computable Linear Orders

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The Dushnik-Miller Theorem for countable linear orders states that any countably infinite linear order \mathcal{L} has a nontrivial self-embedding (i.e., an order-preserving injection f from \mathcal{L} into \mathcal{L}). By a result of Hay and Rosenstein, this theorem does not hold effectively.

An old problem, which has been open for several decades, asks for a characterization of those countably infinite order types τ such that all computable presentations of τ allow a nontrivial computable self-embedding.

It has been conjectured that these order types are exactly those τ such that any infinite interval I of τ admits a fixed finite bound n_I on the size of discrete subintervals of I . Downey has shown that if this conjecture holds, then it cannot hold uniformly, and it is not sufficient to consider only Δ_2^0 -isomorphisms.

Along these lines, I will present two related, joint results:

1. Jointly with Downey, I showed that there is an infinite computable linear order \mathcal{L} such that any nontrivial self-embedding of \mathcal{L} computes the halting problem. As a corollary, we obtain that the Dushnik-Miller Theorem for countable linear orders is equivalent to ACA_0 over RCA_0 .
2. Jointly with McCoy, Morozov and Solomon, I constructed an infinite computable linear order \mathcal{L} without Δ_2^0 -computable nontrivial self-embedding.

The conjecture above remains open and still appears very hard.

Splitting and Nonsplitting in the Computably Enumerable Degrees

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The study of splitting and nonsplitting has been a basic topic since the beginning of the study of the Turing degrees, the results in this area added to a growing impression of structural pathology, and the proof of results in this area provided one of the main technical resources in the computability theory. First we survey the basic results about the splitting and nonsplitting of the c.e. degrees; secondly we introduce some results about the relationship of splitting and nonsplitting and the high/low hierarchy, which are viewed as aids to establishing Turing definability, in particular, we introduce a new nonsplitting theorem of Cooper and Li [ta2]: there exists a low_2 c.e. degree above which $\mathbf{0}'$ is not splittable; finally, we introduce the solution to Lachlan's major subdegree problem, Cooper and Li [ta3] have shown that every c.e. degree $\mathbf{b} \neq \mathbf{0}, \mathbf{0}'$ has a major subdegree. A consequence of this theorem is the continuity of the Harrington nonsplitting base that there is no minimal Harrington nonsplitting base, where a Harrington nonsplitting base is an incomplete c.e. degree above which $\mathbf{0}'$ is not splittable.

The Σ -Definability of Admissible Sets as a Reducibility

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We claim to suggest a natural definition of Turing degrees over admissible ordinals. We discuss the notion of Σ -definability of admissible sets introduced and studied in [1]. In our view, Turing degrees correspond to computational devices given by admissible sets while the Σ -definability of one admissible set in another corresponds to algorithmic reducibility.

In the criterion of Σ -definability for locally countable recursively listed admissible sets we proved in [1], Σ -definable permutations were involved. The new criterion is easier and is applicable to a wider class of admissible sets.

Theorem. *Assume \mathbb{A} and \mathbb{B} are recursively listed admissible sets. Then \mathbb{A} is Σ -definable over \mathbb{B} if and only if each Δ -subset of $o(\mathbb{A})$ is a Δ -subset of $o(\mathbb{B})$.*

Related problems are discussed.

References

- [1] A.S.Morozov, On Σ -definability of admissible sets, In: Proceedings of logic Colloquium-98, Praha, 9-15 August, S. Buss, P. Hájek, P. Pudlák eds. p. 334-351.

Effectively Dense Boolean Algebras and Their Applications

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We use methods from effective algebra to prove the undecidability of the theory of various distributive structures. The structures stem from computability theory, complexity theory and formal logic. We also discuss new results on the Solovay degrees of c.e. reals

Algorithmic Nature of Classical First-Order Predicate Logic

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Sets of integers and integer-valued functions as well as integers themselves are main objects of classical algorithm theory. When we have described an enumeration procedure for a set of integers depending of few external parameters, by s-m-n-theorem (Parameter Theorem), we can effectively in the parameters find a recursively enumerable index of the set of integers resulting. This is some general method to obtain various results in algorithm theory based on the using of the Parameter Theorem.

Constructions of finitely axiomatizable theories can be considered as some kind of Parameter Theorem. When we have described an effective enumeration procedure (depending of some external parameters) for a set of first-order formulas which determines a theory, applying a construction of finitely axiomatizable theory, we can effectively in the parameters find a single first-order formula that (in known sense) has the same model-theoretic properties as the recursively axiomatizable theory we initially constructed. This gives very general and powerful method to obtain particular results about algorithmic properties of formulas in first-order predicate logic.

This idea gives a very natural extension of classical algorithm theory on new (very interesting) objects – first order formulas (which are considered together with their model-theoretic contents).

Resource-Bounded Hausdorff Dimension

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Hausdorff dimension is an important tool in geometric measure theory, especially in the study of fractal objects. It refines Lebesgue measure, thereby allowing to distinguish quantitatively between Lebesgue nullsets. For instance, the well known middle third Cantor set is a set of measure zero having Hausdorff dimension $\log 2 / \log 3$.

In the context of computability, originating from applications of Hausdorff dimension in information theory, a close connection between Hausdorff dimension and Kolmogorov complexity was established. This led to the definition of effective dimension.

Recently, Lutz has extended the theory of effective dimension to complexity theory by introducing resource-bounded dimension. As in the case of resource bounded measure, the resource bound is imposed on the martingales (betting strategies) characterizing the Hausdorff dimension of a class.

This talk focuses on the resource-bounded dimension of objects in structural complexity theory. The main results (among others) are:

1. For every set in E , the dimension of its p - m -degree is the same as the dimension of its p - m -lower span. (This implies that the p - m -complete sets have dimension 1 in E , thereby yielding an example of a class that has measure 0 but dimension 1 in E .)
2. For every Δ_2^0 -computable real $\alpha \in [0, 1]$ there is a set whose p - m -lower span has dimension α in E .

This is joint work with Klaus Ambos-Spies, Wolfgang Merkle, and Frank Stephan from Heidelberg.

Characterising the Non-Total e-Degrees

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We recall that a set A (subset of the set ω of the natural numbers) is enumeration reducible to B ($A \leq_e B$) if there is a uniform algorithm for obtaining the enumeration of the members of A from any given enumeration of B . We denote by $d_e(A)$ the e-degree of A , and by \mathbf{D}_e the upper semilattice of the e-degrees. Let Tot be the subset of \mathbf{D}_e consisted of the total e-degrees. We are interested in two problems about $\mathbf{D}_e \setminus Tot$:

- (i) what can we say about the e-degrees from $\mathbf{D}_e \setminus Tot$ moreover they are non-total;
- (ii) to characterize the non-total e-degrees in terms of properties of some of their elements.

The examination of the first problem resulted in the following

Theorem. *Let \mathbf{b} be any given e-degree then any non-total e-degree $\mathbf{a}=d_e(A)$ such that $\mathbf{b} \leq \mathbf{a} < \mathbf{b}'$ either is a \mathbf{c} -quasi-minimal for certain e-degree $\mathbf{c} \geq \mathbf{b}$ or \mathbf{a} contains the set $E = \bigcup_{s \in \omega} E_s$, where $E_0 <_e E_1 <_e \dots <_e E_s <_e \dots <_e A$ and $d_e(E_s) \in Tot$ for all $s \in \omega$. Incidentally, if $\mathbf{b} \in Tot$ then $d_e(E_0) = \mathbf{b}$.*

The second problem does not have a best possible solution for the present. We give a series of sufficient conditions for non-totality of the e-degrees in terms of the specially chosen classes of sets, of a strong e-reducibility and of T-reducibility.

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Learning How to Separate

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The main question addressed in the present work is how to find effectively a recursive function separating two sets drawn arbitrarily from a given collection of disjoint sets. In particular, it is investigated in which cases it is possible to satisfy the following additional constraints: confidence where the learner converges on all data-sequences; conservativeness where the learner abandons only definitely wrong hypotheses; consistency where also every intermediate hypothesis is consistent with the data seen so far; set-driven learners whose hypotheses are independent of the order and the number of repetitions of the data-items supplied; learners where either the last or even all hypotheses are programs of total recursive functions.

The present work gives an overview of the relations between these notions and succeeds to answer many questions by finding ways to carry over the corresponding results from other scenarios within inductive inference. Nevertheless, the relations between conservativeness and set-driven inference needed a novel approach which enabled to show the following two major results:

1. There is a class for which recursive separators can be found in a confident and set-driven way, but no conservative learner finds a (not necessarily total) separator for this class.
2. There is a class for which recursive separators can be found in a confident and conservative way, but no set-driven learner finds a (not necessarily total) separator for this class.

This is joint work with Sanjay Jain, National University of Singapore

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